# Multicommodity Flow and its Applications(多種フローとその応用)

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# 論文内容要旨

The network flow problem and its variants have been extensively studied. There are efficient algorithms for finding a maximum single-commodity flow. It is also known that one can find the maximum two-commodity flows in an undirected graph by using an algorithm for a single-commodity flow. The situation is different with regard to flows of more than two commodities. No simple polynomial time algorithm has been known for the multicommodity flow problem on general graphs. Therefore simple efficient algorithms are useful in practice even if they are valid for restricted classes of graphs.

The multicommodity flow problem can be applied to many practical problems such as traffic control, design of communication networks, and VLSI routing. The routing problem requires to interconnect terminals on the boundaries of modules by wires. If the problem involves only two-terminal nets, then it can be effectively reduced to the "edge-disjoint path problem", which requires to find edge-disjoint paths, each connecting two terminals specified in a grid. Furthermore the edge-disjoint path problem is reduced to the multicommodity *integral* flow problem. However the edge-disjoint path problem and also multicommodity integral flow problem for general graphs are NP-hard. Therefore *it is very unlikely that there exists a polynomial time algorithm* for solving them. Consequently the routing problem is usually solved by partitioning the given routing region into some regions so that each region has a tractable shape such as L-shaped region. Thus many researchers have studied the edge-disjoint path problem on various classes of gris, and given efficient algorithms for solving the problem. However in these algorithm all the terminals are assumed to lie only on one face boundary.

In this thesis we give efficient algorithms for finding multicommodity flow in three classes of planar graphs and design algorithms for finding edge-disjoint paths in four classes of plane grids. We also develop a new data structure, named a variable-priority queue, and use it in most of our algorithms. The planar graphs and plane grid treated in this thesis are more general than those for which efficient algorithms have been known. For example, in Chapters 7 and 8 we give efficient algorithms for finding edge-disjoint paths in a rectangular grid with one hole. A given routing region can be partitioned into a small number of such grids, and the total computation time needed to find a routing can be reduced. It is expected that the algorithms developed in this thesis can be applied to many practical problems.

We now summarize each chapter. Chapter 1 is Introduction. In Chapter 2 we develop a new data structure called a *variable-priority queue*. The variable-priority queue is a generalization of a queue, but not of a priority queue. The queue supports, in addition to the ordinary queue operations, an operation MIN to find an item of minimum key and operations UPDATE and DECREASE to change keys of items. Any sequence of these $m$ operations can be processed in $O(m)$ time. The data structure is useful in designing efficient algorithms for various network problems such as the multicommodity flow and edge-disjoint path problems on planar graphs. The variable-priority queue is used in many algorithms in this thesis.

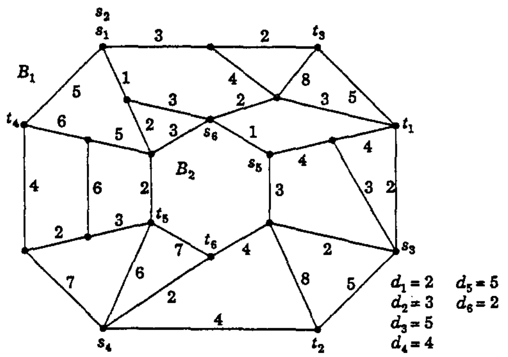


Fig 1. A flow network in $C_{12}$.

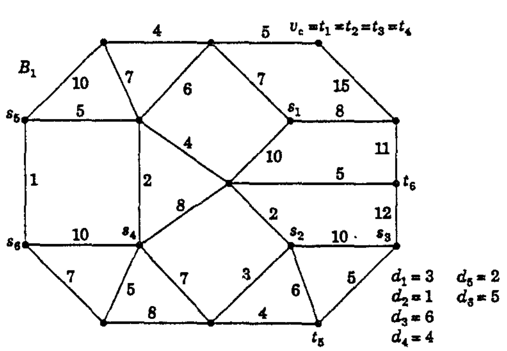


Fig 2. A flow network in $C_{01}$.

In Chapter 3 we present an algorithm for finding multicommodity flows in a cycle network. The algorithm first decides whether a given cycle $G$ has multicommodity flows, each from a source to a sink and of a specified demand, and then acutually finds them if $G$ has. The running time is $O(k+n)$, where $k$ is the number of source-sink pairs and $n$ is the number of vertices in $G$.

In Chapters 4 and 5 we present efficient algorithms for finding multicommodity flows in two classes of planar network $C_{12}$ and $C_{01}$. Every graph in $C_{12}$ has two face boundaries $B_1$ and $B_2$ such that each of the source-sink pairs lies on $B_1$ or $B_2$. (See Fig. 1.) On the other hand, every graph in $C_{01}$ has a face boundary $B_1$ such that some of the source-sink pairs lie on $B_1$ and all the other pairs share a common sink lying on $B_1$. (See Fig. 2.) The multicommodity flow problem for a graph in $C_{12}$ (resp. $C_{01}$) is shown to be reduced to the shortest path problem for an undirected (resp. a directed) graph obtained from the dual of the original undirected graph. The reduction yields simple efficient algorithms for the multicommodity flow problem. The algorithms run in $O(kn+nT(n))$ time if a graph has $n$ vertices and $k$ source-sink pairs and $T(n)$ is the time required for finding the single-source shortest paths in a planar graph of $n$ vertices.

In Chapter 6 we present algorithms for finding edge-disjoint paths in a grid having exactly one nontrivial rectangular hole. All the terminals are assumed to lie on the boundary of the hole. We also present an algorithm for finding a routing using two layers in a given region having a rectangular outer boundary, and an algorithm for finding a routing using three layers in a given region having any shape of the outer boundary. Furthermore an algorithm is presented which finds an outer rectangle of minimum area such that there is a routing using two layers in the region bounded by the outer rectangle and a given inner one. All the algorithms in this chapter run in time linear in the perimeter of the grid.

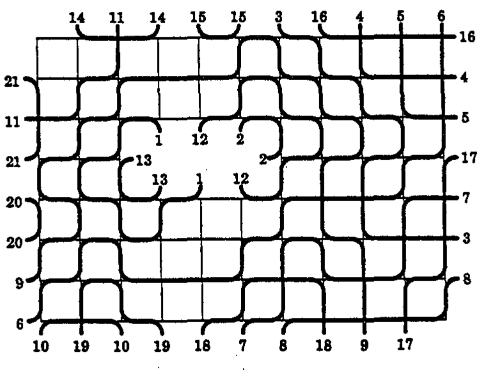


Fig 3. A grid network treated in Chapter 7 and edge-disjoint paths(drawn in thick lines).

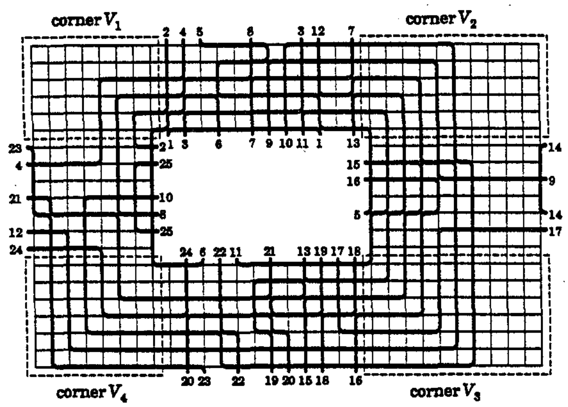


Fig 4. A grid network treated in Chapter 8 and edge-disjoint paths(drawn in thick lines).

In Chapters 7 and 8 we present algorithms for finding edge-disjoint paths in a grid bounded by two nested rectangles. In Chapter 7 we assume that every terminal pair lies on one of the rectangles and, for every vertex, the number of terminals on the vertex plus the number of edges incident with the vertex is an even integer. (See Fig.3.) The algorithm in Chapter 7 runs in $O(n)$ time if a given graph has $n$ vertices. One can solve the prodlem in Chapter 7 by the algorithm in Chapter 4 for finding multicommodity flows in planar glaphs, but then $O(n^2)$ computation time is required. The algorithm in Chapter 7 is entirely different from the flow algorithm. On the other hand, in Chapter 8, we assume that all the terminals lie on the two rectangles, outside of the four rectangular corners. (See Fig. 4.) Note that pairs may have one terminal on the outer rectangle and the other on the inner rectangle. The algorithm in Chapter 8 runs in $O(k^2 \log k)$ time if $k$ is the number of terminal pairs.

In this thesis we presented efficient algorithms for the multicommodity flow and edge-disjoint path problems on several classes of planar graphs and plane grids. However many open problems still remain in the subject. For example, no efficient algorithm has been known for the problem in directed planar graphs. It is expected that efficient algorithms will be found for more general classes of graphs.

# 審 査 結 果 の 要 旨

VLSI配線，通信網構成，交通網制御などの種々の問題はネットワークで多種フローを求める問題に帰着される。多種実数値フロー問題は線形計画問題として定式化できるので，Khachiyanの楕円体法やKarmarkarの射影法を用いれば多項式時間で解けるが，大規模な問題に対しては計算時間がかかり過ぎて実用的でない。また多種整数値フロー問題はNP困難であり，効率の良いアルゴリズムはありそうにない。

著者は平面グラフがVLSIの配線など実用上よく現われることに注目し，平面グラフで多種フローを求める効率の良いアルゴリズムを設計し，それを応用して平面格子グラフで配線を求めるアルゴリズムを与えた。本論文はこの成果をとりまとめたもので，全文9章よりなる。

第1章は序論であり，研究の概要および用語について説明している。

第2章では，可変優先キューという新しいデータ構造を開発し，その効率の良いインプレメンテーションを与えている。この可変優先キューは以下の章のアルゴリズムで用いられる。

第3章では，閉路ネットワークの多種フローを求めるアルゴリズムを与えている。計算時間は線形であるので最適である。

第4章及び第5章では，2種類の平面グラフに対し，多種フローを求める効率の良いアルゴリズムを開発している。第4章では各端子対が平面グラフGの外面あるいは1つの内面のどちらかの上にある場合を扱っている。第5章では，幾つかの端子対は両方の端子がGの外面上にあり，残りの端子対は片方の端子がGの外面上のある1点に固定されている場合を扱っている。どちらの場合も，Gから得られるある種の双対グラフ上で点数に比例する回数だけ最短路問題を解けば多種フローが求まることを示している。このアルゴリズムの計算時間は点数の二乗に比例する程度であり，極めて高速である。

第6,7,8章では，格子グラフで辺素な道を求める線形時間アルゴリズムを与えている。第6章は長方形の穴の周上にだけ端子がある場合，第7章は各端子対が入れ子になった2つの長方形のどちらかの周上にある場合，第8章は端子対が2つの長方形にまたがってもよい場合である。

第9章は結論である。

以上要するに本論文は，平面グラフの多種フローを求める高速なアルゴリズムを開発し，それを応用して格子グラフで辺素な道を求める線形時間アルゴリズムを求めたもので，通信工学および情報工学に寄与するところが少なくない。

よって，本論文は工学博士の学位論文として合格と認める。